

# Active Huygens' Box: Arbitrary Synthesis of EM Waves in Metallic Cavities

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**Abstract**— Previously, it was believed that electromagnetic plane waves cannot exist in a closed metallic cavity. However, by applying the electromagnetic equivalence principle, we have demonstrated the formation of an arbitrary waveform in a region enclosed by an active Huygens' metasurface. In this paper, we apply this principle on two cavity shapes — a rectangular and a cylindrical cavity, which we call the Huygens' boxes. We present simulation results to demonstrate the generation of traveling plane waves at 0° and 45° in both Huygens' boxes. We also present measured experimental results that agrees with the simulation results for the Huygens' box. We have thus demonstrated the generation and control of waveforms in a metallic cavity at an unprecedented level. We believe that this novel discovery will find applications in a wide range of applications like imaging, communication, and medical therapy in the nearest future.

**Keywords**—antennas, diffraction, metasurface, super-resolution, waveform synthesis.

## I. INTRODUCTION

Metasurfaces are surfaces artificially engineered to achieve some desirable electromagnetic behavior, even ones that are unusual or difficult to achieve. Recently, researchers have presented works that validate metasurfaces' excellent capabilities for electromagnetic wave manipulation. These surfaces have been applied to a myriad of applications [1]–[3]. In this paper, we report findings on arbitrary waveform generation in metallic cavities enclosed by active Huygens' metasurfaces. We show through theory, simulation and experiment that, with a concept we call the Huygens' box, we can generate and control electromagnetic waveform to unprecedented levels.

## II. CONCEPT AND THEORETICAL FORMULATION

In [4]–[6] we have demonstrated that it is possible to arbitrarily manipulate a waveform if we enclose a region of interest with a Huygens' metasurface. We call this the Huygens' Box. We invoke the electromagnetic equivalence principle developed by Love and Schelkunoff [7], [8] to find the necessary electric and magnetic currents along the boundary that will generate an arbitrary desired electromagnetic waveform within the region of interest. Then we directly generate the required currents using an active Huygens' metasurface. Fig. 1 gives a general outlook of the Huygens' box concept. □ A set of currents  $J_s$  and  $M_s$  can generate electromagnetic fields  $\{E_a, H_a\}$  within an enclosed area without

affecting  $\{E_b, H_b\}$ , or adjust the electromagnetic field  $\{E_b, H_b\}$  outside the area without affecting  $\{E_a, H_a\}$ . We demonstrate this by way of a simple example. The E-field of a plane wave can be written using the equation:

$$\vec{E} = \vec{E}_0 e^{-j(k(\vec{A}_n \cdot \vec{r}))}, \quad (1)$$

where  $\vec{A}_n$  is a unit vector in the wave propagation direction,  $k$  is the wave number, and  $\eta$  is the wave's intrinsic impedance.

With the provisions of Fig. 1(a) and pre-assuming the fields according to (2);

$$\begin{aligned} \{E_a, H_a\} &= \{E_0 e^{-j(k(A_n \cdot r))}, \frac{1}{\eta} (A_n \times E_b)\} \\ \{E_b, H_b\} &= 0. \end{aligned} \quad (2)$$

We then calculate the required currents using (3).

$$\begin{aligned} J_s &= \hat{n} \times (H_b - H_a) \\ M_s &= -\hat{n} \times (E_b - E_a). \end{aligned} \quad (3)$$

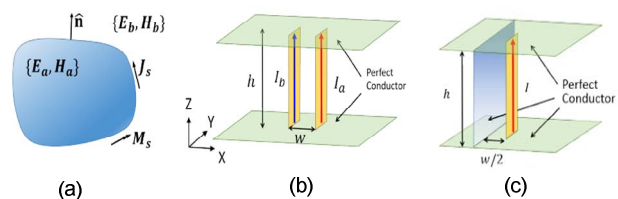


Fig. 1. (a) A diagram showing the equivalence principle where  $\hat{n}$  is the outward-pointing normal,  $\{E_a, H_a\}$  is the electromagnetic field inside the cavity,  $\{E_b, H_b\}$  is the electromagnetic field outside the cavity and  $J_s$  and  $M_s$  are the electric and magnetic surface currents respectively. (b) A simple active Huygens' metasurface element. (c) The mirrored current filament Huygens' metasurface equivalent [5].

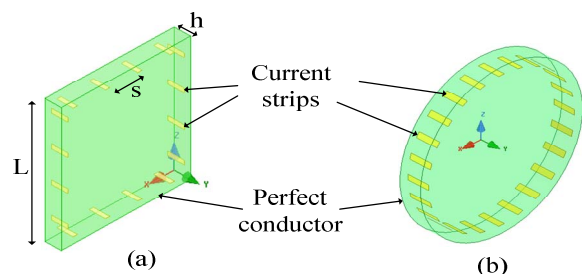


Fig. 2. (a) Diagram of the rectangular Huygens' box. (b) Diagram of the cylindrical Huygens' box

We show the Huygens' boxes in Fig. 2. We implement the Huygens' box using current strips as in Fig. 1(b). Furthermore, we dissect the twin filament of Fig. 1(b) to reduce it to Fig. 1(c) using a perfect electric conductor surface. This serves as an anti-mirror to the single current element, effectively suppressing the electric surface current while doubling the magnetic current. This gives the same effect as the twin current filament setup with fewer current elements [5].

### III. RECTANGULAR CAVITY: SIMULATION AND EXPERIMENTAL RESULTS

Fig. 2(a) shows our rectangular cavity simulation model. We performed full-wave simulation using Ansys HFSS at a simulation frequency of 1GHz. We used 16 active Huygens' sources with separation  $s = 0.25\lambda$  between any two adjacent sources. We generated a Huygens' box with size  $L = \lambda$  in the center of the simulation domain. We present our simulation results in Fig. 3(a-d) showing that upon proper excitation one can excite unconventional waveforms within a rectangular cavity, such as travelling waves, which terminate at the cavity boundaries.

We proceeded to experimentally synthesize and measure the travelling wave (along with several other waveforms) inside the rectangular Huygens' box. Ref. [6] gives a detailed description of our experimental apparatus. We present a comparison between the simulation and the experimental results in Fig. 3(e-h) for a plane wave propagating along the z-axis. The black rectangular frame indicates the movement boundary of the measurement probe used in the experiment. We observe satisfactory agreement in simulation and experiment.

### IV. CYLINDRICAL CAVITY: SIMULATION RESULTS

To further demonstrate the concept of the Huygens' box, we also consider plane wave generation in a cylindrical cavity. The curved geometry attracts our attention, as this geometry is amenable to applications in dual (or circular) polarization aperture antennas and medical imaging modalities. We adapt the same methodology as discussed in the previous section.

Fig. 2(b) shows the simulation model. We performed the simulation at a frequency of 1GHz. Again, we used an array of current strips, backed by a perfect conductor, as the metasurface elements. Each of the 24 equidistantly-spaced current strips has a width of  $\lambda/10$ . We present the simulation results in Fig. 4. It shows plane waves generated within the cavity travelling at  $0^\circ$  and  $45^\circ$  from the horizontal axis at different phase intervals.

### V. CONCLUSION

In this paper, we have demonstrated the synthesis of plane waves using active Huygens' boxes of two different geometries. Such waveforms would not have been formed in typical metallic cavities. One can synthesize waveforms that are more complicated by simply superimposing the constituent plane waves demonstrated in this paper. Our work hence opens doors to many applications through generating arbitrary

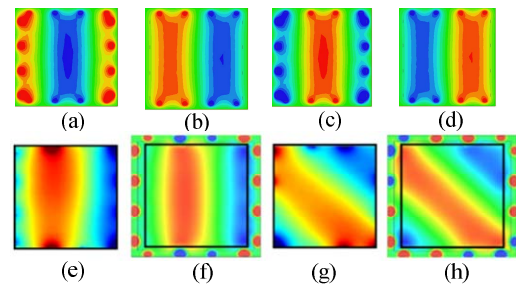


Fig. 3. Generating plane waves inside the Rectangular Huygens' Box. (a-d) Plane wave propagating at  $0^\circ$  from the z-axis at four phase instances. (e) Measured and (f) simulated electric field (real part, at one phase-point) inside the Huygens' box for the  $0^\circ$  plane wave. (g-h) The same for  $45^\circ$  plane wave. The black boxes indicate regions reachable by the probe.

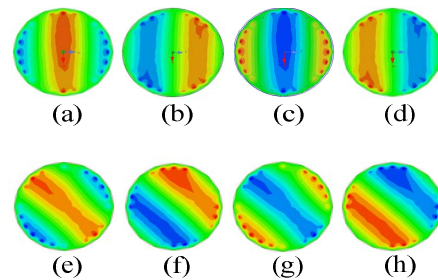


Fig. 4. Generating plane waves inside the Cylindrical Huygens' Cavity. (a-d) Plane wave propagating at  $0^\circ$  (horizontal direction) at four phase instances. (e-h) Plane wave propagating at  $45^\circ$  (from the horizontal axis) at four phase instances.

waveforms in closed or partially closed cavities. We expect that this new metasurface device shall find a myriad of applications in imaging, electromagnetic mode management and medical therapy.

### REFERENCES

- [1] C. L. Holloway, E. F. Kuester, J. A. Gordon, J. O'Hara, J. Booth, and D. R. Smith, "An overview of the theory and applications of metasurfaces: the two-dimensional equivalents of metamaterials," *IEEE Antennas Propag. Mag.*, vol. 54, no. 2, pp. 10-35, Apr. 2012.
- [2] A. Epstein and G. V. Eleftheriades, "Huygens' metasurfaces via the equivalence principle: design and applications," *J. Opt. Soc. Am. B*, 33 A31-A50 (2016).
- [3] N. Yu and F. Capasso, "Light propagation with phase discontinuities: generalized laws of reflection and refraction," *Science*, vol. 334, pp. 333-337, Oct. 2011.
- [4] A. M. H. Wong and G. V. Eleftheriades, "Active Huygens' box: metasurface-enabled arbitrary electromagnetic wave generation inside a cavity," *IEEE International Symposium on Antennas and Propagation (AP-S)*, Boston, USA, Jul. 2018.
- [5] A. M. H. Wong and G. V. Eleftheriades, "A simple active Huygens' source for studying waveform synthesis with Huygens' metasurfaces and antenna arrays," *IEEE Int. Symp. Antennas Propag.*, Jul. 2015.
- [6] A. M. H. Wong and G. V. Eleftheriades, "Active Huygens' metasurfaces for RF waveform synthesis in a cavity," *IEEE Mediterranean Electrotechnical Conf. (MELECON)*, Apr. 2016.
- [7] A. E. H. Love, "The integration of the equations of propagation of electric waves," *Phil. Trans. Roy. Soc. London, Ser. A* 197, 1-45 (1901).
- [8] S. A. Schelkunoff, "Some equivalence theorems of electromagnetics and their application to radiation problems," *Bell Syst. Tech. J.* 15, 92-112 (1936).